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VERTEX-DISJOINT TRIANGLES IN CLAW-FREE GRAPHS WITH MINIMUM DEGREE AT LEAST THREE

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A graph is said to be claw-free if it does not contain an induced subgraph isomorphic to $K_{1,3}$. Our main result is as follows: For any integer $k \geq 2$, if G is a claw-free graph of order at least $6(k-1)$ and with minimum degree at least 3, then G contains k vertex-disjoint triangles unless G is of order $6(k-1)$ and G belongs to a known class of graphs. We also construct a claw-free graph with minimum degree 3 on n vertices for each $3k \leq n < 6(k-1)$ such that it does not contain k vertex-disjoint triangles. We put forward a conjecture on vertex-disjoint triangles in $K_{1,t}$-free graphs.

1. Introduction

A graph is said to be claw-free if it does not contain an induced subgraph isomorphic to $K_{1,3}$. Let G be a claw-free graph of order n with minimum degree at least 3. Clearly, every vertex of degree at least 3 in G is contained in a triangle. We wonder how many vertex-disjoint triangles G contains. Given an integer $k \geq 2$, it is easy to show that if n is sufficiently large (for instance, it suffices to take $n > 18(k-1)$), then G contains k vertex-disjoint triangles. Our problem is to determine the least integer n_k such that when $n > n_k$ then G contains k vertex-disjoint triangles. We will show that $n_k = 6(k-1)$. To state the result, we define the following two graphs F and W.

Let W be a wheel of order 6, i.e., W is obtained from a cycle of length 5 by adding a new vertex to the cycle such that the new vertex is adjacent to every vertex of the cycle. Let F be the graph of order 6 obtained from a cycle of length 6 by adding three new edges to the cycle such that the three new edges form a triangle in F. We use $C(F)$ to denote the set of three vertices of degree 4 in F and let $P(F) = V(F) - C(F)$. We also use $C(F)$ to denote the triangle of F induced by $C(F)$.

For a graph G and a subset $X \subseteq V(G)$, we use $G[X]$ to denote the subgraph of G induced by X. Let $\sum_k$ be the set of graphs of order $6(k-1)$ such that a graph G belongs to $\sum_k$ if and only if each component of G is either isomorphic to W or obtained from $2t$ vertex-disjoint copies F_1, F_2, …, F_{2t} of F for some $t \geq 1$ by adding

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$3t$ new edges so that F_i is an induced subgraph of G for all $i \in \{1,2,\ldots,2t\}$ and so that the new edges form a perfect matching on $\cup_{i=1}^{2t} P(F_i)$. It is easy to see that if $G \in \sum_k$ then each triangle of G is contained in an induced subgraph of G that is isomorphic to W or F. Hence if $G \in \sum_k$ and $|V(G)| = 6(k-1)$, then $G \supseteq (k-1)K_3$ and $G \not\supseteq kK_3$.

In this paper, we prove the following result.

Theorem A. *Let $k \geq 2$ be an integer and G a claw-free graph of order at least $6(k-1)$ and with minimum degree at least 3. Then G contains k vertex-disjoint triangles unless G belongs to $\sum_k$.*

It is easy to see that for integers $k \geq 2$ and $3k \leq n < 6(k-1)$, there is a claw-free graph H_n of order n and with minimum degree at least 3 such that each component of H_n is isomorphic to either K_4, or K_5, or a graph in $\sum_t$ for some $t \geq 2$. We choose H_n such that the number of components isomorphic to K_4 or K_5 is minimal. Then it is easy to see that H_n does not contain k vertex-disjoint triangles.

Claw-free graphs have been investigated much in hamiltonian graph theory. For some results on this topic, see [5, 6, 7, 12]. Las Vergnas [4] and Sumner [8] proved that a connected claw-free graph of order $2k$ contains a perfect matching, i.e., k vertex-disjoint copies of K_2. Our work is an extension in this direction to determine at least how many vertex-disjoint triangles a claw-free graph contains. About vertex-disjoint triangles in a general graph, we would like to mention a result of Corrádi and Hajnal [2]. They proved that if G is a graph of order at least $3k$ and with minimum degree at least $2k$, then G contains k vertex-disjoint cycles. In particular, when the order of G is exactly $3k$, then G contains k vertex-disjoint triangles. A generalization of this result is given in [11], that is, if $d(x)+d(y) \geq 4k-1$ for each pair of non-adjacent vertices x and y of G, then G contains k vertex-disjoint cycles.

We conclude our introduction by putting forward a conjecture on vertex-disjoint triangles in $K_{1,t}$-free graphs. A $K_{1,t}$-free graph is a graph with no induced subgraphs isomorphic to $K_{1,t}$. Let $h(t,k)$ be the smallest integer such that every $K_{1,t}$-free graph of order greater than $h(t,k)$ and with minimum degree at least t contains k vertex-disjoint triangles. By Theorem A, $h(3,k) = 6(k-1)$ for $k \geq 2$. We conjecture the following.

Conjecture B. *For each integer $t \geq 4$, there exists an integer k_t depending on t only such that $h(t,k) = 2t(k-1)$ for all integers $k \geq k_t$.*

We discuss only finite simple graphs and use standard terminology and notation from [1] except as indicated. Let G be a graph. If u is a vertex of G and H is either a subgraph of G or a subset of $V(G)$, we define $N(u,H)$ to be the set of neighbors of u contained in H, i.e., $N(u,H) = N(u) \cap V(H)$ or $N(u,H) = N(u) \cap H$, respectively. We let $d(u,H) = |N(u,H)|$. Thus $d(u,G)$ is the degree of u in G. If H' is also a subgraph of G, we define $N(H',H) = \cup_{x \in V(H')} N(x,H)$. For a subset U of $V(G)$, $G[U]$ denotes the subgraph of G induced by U. When $U = \{x_1, x_2, \ldots, x_t\}$, we may also use $G[x_1,x_2,\dots,x_t]$ to denote $G[\{x_1,x_2,\dots,x_t\}]$. If there is no confusion, we use $[U]$ for $G[U]$. If S is a set of subgraphs of G, we write $G\supseteq S$. We shall use mK_3 to represent a set of m vertex-disjoint triangles. For two vertex-disjoint subgraphs G_1 and G_2 of G, $e(G_1,G_2)$ is the number of edges of G between G_1 and G_2. If G' is a graph and we write $G'\subseteq G$ or $G\supseteq G'$, it means that G' is isomorphic to a subgraph of G.

2. Lemmas

First, we define two graphs B and D. Let B be the graph of order 6 obtained from K_4 and a path P of order 4 such that the two end vertices of P are in K_4. Let D be the graph of order 5 consisting of two triangles which have a common vertex. In the following, $G=(V,E)$ is a claw-free graph.

Lemma 2.1. *Let H be an induced subgraph of order 6 in G and $u\in V-V(H)$. Then the following two statements hold.*

(*a*) *If $H\cong W$ and $d(u,H)>0$ then $H+u\supseteq 2K_3$.*

(*b*) *If $H\cong F$ and $d(u,C(H))>0$ then $H+u\supseteq 2K_3$.*

Proof. First, suppose $H\cong W$. Let x_0 be the vertex of H with $d(x_0,H)=5$. Then we must have $d(u,H-x_0)>0$ for otherwise $[x_0,u,v,w]\cong K_{1,3}$ where $\{v,w\}\subseteq N(x_0,H)$ with $v\neq w$ and $vw\notin E$, a contradiction. Let $x\in V(H-x_0)$ be such that $ux\in E$ and $\{y,z\}=N(x,H-x_0)$. As it is claw-free, $[x,u,y,z]\supseteq K_3$. Clearly, $H-\{x,y,z\}$ is a triangle. Thus $H+u\supseteq 2K_3$. So (*a*) holds. The proof of (*b*) is evident. ∎

Lemma 2.2. *Let H be an induced subgraph of order 6 in G such that either $H\supseteq F$ or $H\supseteq W$. If $H\not\supseteq 2K_3$ then $H\cong F$ or $H\cong W$.*

Proof. Evident. ∎

Lemma 2.3. *Let H_1 and H_2 be two vertex-disjoint induced subgraphs of G such that either $H_i\cong F$ or $H_i\cong W$ for every $i\in\{1,2\}$. Then the following two statements hold.*

(*a*) *If $H_1\cong W$ and $e(H_1,H_2)>0$ then $[V(H_1\cup H_2)]\supseteq 3K_3$.*

(*b*) *If $H_1\cong H_2\cong F$ and $e(C(H_1),H_2)>0$ then $[V(H_1\cup H_2)]\supseteq 3K_3$.*

Proof. First, suppose $H_1\cong W$. Let $x_0\in V(H_1)$ with $d(x_0,H_1)=5$. As in the proof of Lemma 2.1, it is easy to deduce that $d(u,H_1-x_0)>0$ for some $u\in V(H_2)$. Say $uv\in E$ for a vertex $v\in V(H_1-x_0)$. If $H_2\cong F$ with $u\in C(H_2)$ or $H_2\cong W$, then by Lemma 2.1, $H_2+v\supseteq 2K_3$, and so $[V(H_1\cup H_2)]\supseteq 3K_3$ as $H_1-v\supseteq K_3$. If $H_2\cong F$ and $u\in P(H_2)$, then by Lemma 2.1, $H_1+u\supseteq 2K_3$, and so $[V(H_1\cup H_2)]\supseteq 3K_3$ as $H_2-u\supseteq K_3$. So (*a*) holds.

Next, suppose $H_1\cong H_2\cong F$. As G is claw-free, it is easy to deduce that there exists $u\in P(H_2)$ such that $d(u,C(H_1))>0$. By Lemma 2.1, $H_1+u\supseteq 2K_3$ and so $[V(H_1\cup H_2)]\supseteq 3K_3$. Thus (*b*) holds, too. ∎

Lemma 2.4. *Let H be an induced subgraph of order 5 in G such that $H \supseteq D$. Let $x_0 \in V - V(H)$ be such that $d(x_0, H) > 0$. Then there exists $b \in V(H)$ such that $x_0 b \in E$ and $H - b \supseteq K_3$.*

Proof. Let $V(H) = \{x, y, z, u, v\}$ be such that $H \supseteq \{xyzx, xuvx\}$. Clearly, the lemma is true if $d(x_0, H - x) > 0$. So assume $x_0 x \in E$ and $d(x_0, H - x) = 0$. Then $\{uy, vy\} \subseteq E(H)$ as $[x, x_0, u, y] \ncong K_{1,3}$ and $[x, x_0, v, y] \ncong K_{1,3}$. Thus $yuvy$ is a triangle in $H - x$. ∎

Lemma 2.5. *Let H be an induced subgraph of order 5 in G such that $H \supseteq D$. Let x be the common vertex of two edge-disjoint triangles in H. Let $x_0 \in V - V(H)$ be such that $d(x_0, H) \geq 2$ and $x_0 x \in E$. Then either $H + x_0 \supseteq 2K_3$ or $H + x_0 \cong W$.*

Proof. Let $V(H) = \{x, y, z, u, v\}$ be such that $H \supseteq \{xyzx, xuvx\}$ and $\{x_0 x, x_0 u\} \subseteq E$. Suppose $H + x_0 \not\supseteq 2K_3$. Then we see that $x_0 v \notin E$ and $d(x_0, yz) \leq 1$. Say $x_0 y \notin E$. Then $vy \in E$ as $[x, x_0, v, y] \ncong K_{1,3}$. Thus $vz \notin E$ for otherwise $H + x_0 \supseteq 2K_3 = \{x_0 u x x_0, vyzv\}$. Then $x_0 z \in E$ as $[x, x_0, v, z] \ncong K_{1,3}$. Hence $H + x_0 \supseteq W$. By Lemma 2.2, $H + x_0 \cong W$. ∎

Lemma 2.6. *Let T be a triangle in G and S a set of three distinct vertices in $N(T, G - V(T))$. Let $H = [V(T) \cup S]$. Suppose $d(b, H) \geq 2$ for all $b \in S$. Then either $H - b \supseteq D$ for some $b \in S$, or $H \supseteq 2K_3$, or $H \cong B$.*

Proof. Suppose that $H - b \not\supseteq D$ for all $b \in S$ and $H \not\supseteq 2K_3$. We shall prove $H \cong B$. If $d(b, T) \geq 2$ for all $b \in S$, then it is easy to see that $H - b \supseteq D$ for some $b \in S$. So we may assume $\{uv, ux\} \subseteq E$ and $d(u, T) = 1$ where $S = \{u, v, w\}$ and $T = xyzx$. Thus $vx \notin E$ as $H - w \not\supseteq D$. Say $vy \in E$.

If $uw \in E$, then $vw \notin E$ as $H \not\supseteq 2K_3$ and $wx \notin E$ as $H - v \not\supseteq D$, and consequently, $[u, v, w, x] \cong K_{1,3}$, a contradiction. Hence $uw \notin E$. Similarly, we must have $vw \notin E$. If $wx \in E$ then $wy \in E$ as $[x, u, w, y] \ncong K_{1,3}$. Similarly, if $wy \in E$ then $wx \in E$. As $d(w, H) \geq 2$, we see that $\{x, y\} \cap N(w) \neq \emptyset$ and therefore $\{x, y\} \subseteq N(w)$. Then $wz \in E$ as $[x, u, w, z] \ncong K_{1,3}$. Then $vz \notin E$ as $H - u \not\supseteq D$. Hence $H \cong B$. ∎

Lemma 2.7. *Let T be a triangle in G and S a set of four distinct vertices in $N(T, G - V(T))$. Then either there exists $\{u, v\} \subseteq S$ such that $[V(T) \cup \{u, v\}] \supseteq D$, or there exists $u \in S$ such that $T + u \cong K_4$.*

Proof. Suppose that there exists no $\{u, v\} \subseteq S$ such that $[V(T) \cup \{u, v\}] \supseteq D$. Then for every $b \in V(T)$, $N(b, S)$ does not contain two adjacent vertices. As G is claw-free, this implies that $d(b, S) \leq 2$ for every $b \in S$. Hence there exists $x \in V(T)$ such that $d(x, S) = 2$. Let $T = xyzx$ and $N(x, S) = \{u, v\}$. As $uv \notin E$ and $[x, u, v, y] \ncong K_{1,3}$, we may assume $uy \in E$. If $vz \in E$, then $[V(T) \cup \{u, v\}] \supseteq D$, a contradiction. Thus $vz \notin E$ and $uz \in E$ as $[x, u, v, z] \ncong K_{1,3}$. Hence $T + u \cong K_4$. ∎

3. Proof of Theorem A

Let $k \geq 2$ be an integer and $G=(V,E)$ a claw-free graph of order $n \geq 6(k-1)$ and with $\delta(G) \geq 3$. Suppose that G does not contain k vertex-disjoint triangles. We shall prove $G \in \sum_k$. Let s be the greatest integer in $\{1,2,\ldots,k-1\}$ such that G contains s vertex-disjoint triangles. Let r be the greatest integer in $\{1,2,\ldots,s\}$ such that G contains s vertex-disjoint induced subgraphs of order at most 5, say F_1, F_2, …, F_r and T_{r+1}, T_{r+2}, …, T_s, such that for each $i \in \{1,2,\ldots,r\}$, either $F_i \supseteq D$ or $F_i \cong K_4$, and for each $j \in \{r+1,r+2,\ldots,s\}$, T_j is a triangle. Subject to this, we choose these induced subgraphs such that $|V(\cup_{i=1}^r F_i)|$ is maximal.

Let $Q=\cup_{j=r+1}^s T_j$ and $R=G-V((\cup_{i=1}^r F_i)\cup Q)$. Let $U=\{u_1,u_2,\ldots,u_p\}$ be the list of vertices of R with $d(u_i,Q)=0$ for all $i \in \{1,2,\ldots,p\}$. Set $R_0=R-U$. We shall prove the following two claims.

Claim 1. *There exists an injection σ: $\{1,2,\ldots,p\} \to \{1,2,\ldots,r\}$ such that either $F_{\sigma(i)}+u_i \cong F$ or $F_{\sigma(i)}+u_i \cong W$ for all $i \in \{1,2,\ldots,p\}$.*

Proof of Claim 1 Let $i \in \{1,2,\ldots,p\}$. First, suppose $d(u_i,F_j) \leq 1$ for all $j \in \{1,2,\ldots,r\}$. By Lemma 2.4, we can choose a set S of three distinct neighbors of u_i such that $F_j - S \supseteq K_3$ for all $j \in \{1,2,\ldots,r\}$. As it is claw-free, $[S \cup \{u_i\}] \supseteq K_3$ and therefore $G \supseteq (s+1)K_3$, a contradiction. Hence $d(u_i,F_{\sigma(i)}) \geq 2$ for some $\sigma(i) \in \{1,2,\ldots,r\}$. We show that $F_{\sigma(i)}+u_i \cong F$ or W, and there is no vertex u in $U-\{u_i\}$ such that $d(u,F_{\sigma(i)}) \geq 2$. If $F_{\sigma(i)} \cong K_4$ then $F_{\sigma(i)}+u_i \supseteq D$, contradicting the maximality of $|V(\cup_{j=1}^r F_j)|$. Therefore $F_{\sigma(i)} \supseteq D$. Let $V(F_{\sigma(i)})=\{x,y,z,v,w\}$ be such that $F_{\sigma(i)} \supseteq \{xyzx, xvwx\}$. As $F_{\sigma(i)}+u_i \not\supseteq 2K_3$, we see that $d(u_i,vw) \leq 1$ and $d(u_i,yz) \leq 1$. If $u_ix \in E$, then by Lemma 2.5, $F_{\sigma(i)}+u_i \cong W$, and so $N(F_{\sigma(i)}+u_i, R-\{u_i\})=\emptyset$ by Lemma 2.1. So we may assume $d(u_i,F_{\sigma(i)})=2$ and $N(u_i,F_{\sigma(i)})=\{v,y\}$. Let b be a neighbor of u_i with $b \notin \{v,y\}$. If $b \in V(F_j)$ for some $j \in \{1,2,\ldots,r\}$, then by Lemma 2.4, we can choose $b \in V(F_j)$ such that $F_j - b$ contains a triangle. Hence $F_{\sigma(i)}+u_i+b \not\supseteq 2K_3$ as $G \not\supseteq (s+1)K_3$. As $[u_i,b,v,y] \ncong K_{1,3}$, this implies that $bv \notin E$, $by \notin E$ and $vy \in E$. Hence $F_{\sigma(i)}+u_i \supseteq F$, and so $F_{\sigma(i)}+u_i \cong F$ by Lemma 2.2. If there is another vertex c in R such that $c \neq u_i$ and $N(c) \cap \{v,x,y\} \neq \emptyset$, then $F_{\sigma(i)}+u_i+c \supseteq 2K_3$ by Lemma 2.1, a contradiction. Hence $N(v,R)=N(y,R)=\{u_i\}$ and $N(x,R)=\emptyset$. This argument allows us to see that if there is another vertex u' in U such that $u' \neq u_i$ and $d(u',F_{\sigma(i)}) \geq 2$, then $N(u',F_{\sigma(i)})=\{w,z\}$ and $wz \in E$ and so $F_{\sigma(i)}+u_i+u' \supseteq 2K_3$, a contradiction. Hence σ is an injection from $\{1,2,\ldots,p\}$ to $\{1,2,\ldots,r\}$. This shows the claim. ∎

We may assume that $\sigma(i)=i$ and let $G_i=F_i+u_i$ for all $i \in \{1,2,\ldots,p\}$.

Claim 2. $p=r=s=k-1$.

Proof of Claim 2. By Lemma 2.7 and the maximality of r, we see that $|N(T_i,R_0)|\leq 3$ for all $i\in\{r+1,r+2,\ldots,s\}$, and so $|V(R_0)|\leq 3(s-r)$. As $|V(\cup_{i=1}^r F_i)|\leq 5r$ and $n\geq 6(k-1)$, we must have $p\geq r$. As $p\leq r$ by Claim 1, it follows that

(1) $$p=r \text{ and } s=k-1;$$

(2) $$|N(T_i,R_0)|=3 \text{ for all } i\in\{r+1,r+2,\ldots,s\};$$

(3) $$N(T_i,R_0)\cap N(T_j,R_0)=\emptyset \text{ for all } r+1\leq i<j\leq s.$$

By Lemma 2.1 and the maximality of s, we have

(4) $$d(u,G_i)=0 \text{ if } G_i\cong W \text{ and } d(u,C(G_i))=0 \text{ if } G_i\cong F \text{ for all } u\in V(R_0) \text{ and } i\in\{1,2,\ldots,r\}.$$

On the contrary, suppose $r<s$. Let $j\in\{r+1,r+2,\ldots,s\}$ and $G_j=[V(T_j)\cup N(T_j,R_0)]$. By the maximality of r and s, $G_j\not\supseteq K_4$, $G_j\not\supseteq D$ and $G_j\not\supseteq 2K_3$.

Write $T_j=xyzx$ and $N(T_j,R_0)=\{u,v,w\}$. If $d(b,G_j)\geq 2$ for all $b\in N(T_j,R_0)$, then by Lemma 2.6, $G_j\supseteq K_4,D$ or $2K_3$, a contradiction.

Therefore we may assume $d(u,G_j)=1$ and $ux\in E$. As G_j is claw-free and $G_j\not\supseteq D$, we see that either $vx\notin E$ or $wx\notin E$, say $vx\notin E$. Then we may assume $vy\in E$.

Let b_1 and b_2 be two distinct neighbors of u with $x\notin\{b_1,b_2\}$. By (3) and (4), $G_j+b_1+b_2\not\supseteq 2K_3$ for otherwise $G\supseteq(s+1)K_3=kK_3$. Hence $b_1b_2\notin E$. As $[u,b_1,b_2,x]\not\cong K_{1,3}$, we may assume $b_1x\in E$. Thus $G_j+b_1\supseteq D$. By the maximality of r and (4), we must have $b_1\in P(G_i)$ for some $i\in\{1,2,\ldots,r\}$ with $G_i\cong F$. Say $V(G_i)=\{y_1,y_2,\ldots,y_6\}$ with $G_i\supseteq\{y_1y_2\ldots y_6y_1,y_2y_4y_6y_2\}$ and $y_1=b_1$. If $wy\in E$ then it is easy to see that $G_j+b_1\supseteq 2K_3$ for $[y,v,w,z]\not\cong K_{1,3}$ and so $[y,v,w,z]\supseteq K_3$, a contradiction. Hence $wy\notin E$. It follows that $wx\notin E$ for otherwise $[x,u,w,y]\cong K_{1,3}$. Therefore $wz\in E$. If $b_1w\in E$, then since G is claw-free, we get $[b_1,u,w,y_2]\supseteq K_3$, which implies $[V(G_j)\cup V(G_i)]\supseteq 3K_3$ because $[V(G_j)\cup V(G_i)]-\{b_1,u,w,y_2\}\supseteq\{xyzx,y_4y_5y_6y_4\}=2K_3$, a contradiction. Thus $b_1w\notin E$. If $vw\in E$, let w' be a neighbor of w with $w'\notin\{v,z\}$. Then by (3) and (4), we see that $G_j+b_1+w'\not\supseteq 2K_3$ for otherwise $G\supseteq(s+1)K_3=kK_3$. But as it is claw-free, $[w,v,z,w']\supseteq K_3$, and so $G_j+b_1+w'\supseteq 2K_3$, a contradiction. Hence we must have $d(w,G_j)=1$. Then similarly, we can show that there exists $c_1\in P(G_t)$ for some $t\in\{1,2,\ldots,r\}$ with $G_t\cong F$ such that $c_1w\in E$ and $c_1z\in E$. Therefore $G_j+b_1+c_1\supseteq 2K_3$, and consequently, $G\supseteq kK_3$, a contradiction. This shows Claim 2. ∎

We are ready to complete the proof. Let $i\in\{1,2,\ldots,k-1\}$. By the two claims and Lemma 2.3, we see that, if $G_i\cong W$ then G_i is a component of G, and if $G_i\cong F$ then $e(C(G_i),G-V(G_i))=0$. Furthermore, we see that if $G_i\cong F$, then

$d(u,G-V(G_i))=1$ for every $u\in P(G_i)$. To see this, say instead $d(u,G-V(G_i))\geq 2$ for some $u\in P(G_i)$ and let u_1,u_2 and u_3 be three distinct neighbors with $u_1\in C(G_i)$ and $\{u_2,u_3\}\subseteq V-V(G_i)$. Then $[u,u_1,u_2,u_3]\supseteq K_3$ as it is claw-free and so $G_i+u_2+u_3\supseteq 2K_3$, and therefore $G\supseteq kK_3$, a contradiction. This completes the proof of the theorem. ∎

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